This is alpha equation:

$$\begin{split} &\frac{\partial \alpha_{1}}{\partial t} + \nabla \cdot \left(\mathbf{U}\alpha_{1}\right) + \nabla \cdot \left[\mathbf{U}_{\mathbf{r}}\alpha_{1}\left(1 - \alpha_{1}\right)\right] \\ &= \alpha_{1}\dot{\mathbf{v}}_{\nu\alpha} + \left(1 - \alpha_{1}\right)\dot{\mathbf{v}}_{c\alpha} = \left(\dot{\mathbf{v}}_{\nu\alpha} - \dot{\mathbf{v}}_{c\alpha}\right)\alpha_{1} + \dot{\mathbf{v}}_{c\alpha} \\ &= S_{p}\alpha_{1} + S_{u} \end{split}$$

ABOUT Explicitly solving alpha equation:

$$\begin{split} &\frac{\partial \alpha_{1}}{\partial t} + \nabla \cdot \left(\mathbf{U}\alpha_{1}\right) + \nabla \cdot \left[\mathbf{U}_{\mathbf{r}}\alpha_{1}\left(1 - \alpha_{1}\right)\right] \\ &= \left(\dot{v}_{v\alpha} - \dot{v}_{c\alpha}\right)\alpha_{1} + \dot{v}_{c\alpha} = S_{p}\alpha_{1} + S_{u} \end{split}$$

Discretize the equation as:

$$\frac{\alpha_{1} - \alpha_{1}^{0}}{\Delta t} + \frac{\int \nabla \cdot (\mathbf{U}\alpha_{1})dV}{\Delta V} + \frac{\int \nabla \cdot \left[\mathbf{U}_{r}\alpha_{1}(1 - \alpha_{1})\right]dV}{\Delta V} = S_{p}\alpha_{1} + S_{u}$$

And explicit solution is [this is what is done in function of MULE::explicitSolve()]

$$\alpha_{1} = \frac{\alpha_{1}^{0}}{\Delta t} + S_{u} - \frac{\int \nabla \cdot (\mathbf{U}\alpha_{1})dV}{\Delta V} + \frac{\int \nabla \cdot \left[\mathbf{U_{r}}\alpha_{1}(1-\alpha_{1})\right]dV}{\Delta V} + \frac{1}{\Delta V}$$

[redundant divU?

[bug: divU should not appear in Su term]